Math 147 September 24 Worksheet

Problems denoted with (*) are to be placed in your math diary.

1. Calculate the given derivatives for the following functions using the chain rule.

   (a) Find $\frac{\partial f}{\partial r}$ for $f(x, y) = xy$ with $x = r\cos(\theta)$, and $y = r\sin(\theta)$.

   (b) (*) Find $\frac{\partial f}{\partial t}$ for $f(x, y) = x/y$. As we are leaving x, y in general, writing $x' = dx/dt$ and $y' = dy/dt$ may help your calculation. After fully simplifying, what major Calc 1 fact have you proven?

2. Find the directional derivatives of the following functions at the given point and along the given vector.

   (a) (*) $f(x, y) = xy, P = (0, -2)$, along the vector $\mathbf{v} = \frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}$.

   (b) $f(x, y, z) = \frac{1}{x^2+y^2+z^2}$, $P = (2, 3, 1)$, along $\mathbf{v} = \frac{1}{\sqrt{6}}\mathbf{i} + \frac{1}{\sqrt{6}}\mathbf{j} - \frac{2}{\sqrt{6}}k$.

3. (*) Follow the steps to calculate the tangent plane for the given function.

   (a) Consider the level surface $f(x, y, z) = xe^y\cos(z) - z = 1$. Find the gradient of this function at $P = (1, 0, 0)$.

   (b) Find an equation for the tangent plane to the surface at P.

   (c) Rewrite your equation so it is explicit in z. That is, something like $z = f(x, y) = \ldots$.

4. (Gradient Descent) The gradient has applications in optimization (in particular machine learning) as well. Discuss with your group members about the following questions to learn how it is used.

   (a) Recall that in Monday's class, we learned that for the function $f(x, y)$, the directional derivative $D_uf(a, b)$ is at its largest when u is pointing along the gradient $\nabla f(a, b)$. If we were standing on the surface given by $f(x, y)$, in which direction should we move in order to have the least (could be negative) movement?

   (b) Suppose we were on the edge of a valley, such as $z = \frac{x^2}{4} + y^2$. We want to get to the *lowest* point of the valley, i.e. the minimum. Come up with a strategy to get to the bottom of the valley just by taking one step at a time. To be precise, if we are at the point (a, b), where should our next step take us? (Hint: Which direction should we move?)

   (c) While we already know how to find minima of simple functions, this method is particularly useful for functions in which we cannot solve $f_x = f_y = 0$, which arise often in application. However, there are some drawbacks. Try to think up some scenarios in which this process would fail to get to a minimum. In addition, when will we get to an absolute minimum?

   (d) Graph the function $z = \sin(x) + \sin(y)$. Can you foresee any problems using our strategy with this function?